# Exercise I
# Continuum mechanics

## 1 3D strain→stress relations

We can derive the strain→stress relations, from the 3D stress→strain:

$$\varepsilon_{ij} = \frac{(1+\nu)}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} \tag{1}$$

$$\sigma_{ij} = \frac{E}{1+\nu}\varepsilon_{ij} + \frac{\nu}{1+\nu}\sigma_{kk}\delta_{ij}$$

To express the term $\sigma_{kk}$ in function of the strain, we introduce the mean stress and the volumetric strain:

$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$(\sigma_x + \sigma_y + \sigma_z) = \frac{E}{1-2\nu}\varepsilon_v$$

Called $\kappa = \frac{E}{3(1-2\nu)}$ the bulk modulus, we can write

$$\sigma_m = \kappa\varepsilon_v$$

Considering the previous equations now we can express $\sigma_{ij}$ as

$$\sigma_{ij} = \frac{E}{1+\nu}\varepsilon_{ij} + \frac{\nu}{1+\nu}\left(\frac{E}{1-2\nu}\varepsilon_v\right)\delta_{ij}$$

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_v\delta_{ij}\right) \tag{2}$$

## 2 Relations in 2D plain stress problem

In 2D plain stress problem, the stress in one dimension is equal to zero:

$$\sigma_z = 0$$

Considering the general stress→strain expression in 3D domain (1), this reduces to

$$\varepsilon_{ij} = \frac{(1+\nu)}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij}$$

$$\varepsilon_{ij} = \frac{(1+\nu)}{E}\sigma_{ij} - \frac{\nu}{E}(\sigma_x + \sigma_y)\delta_{ij} \tag{3}$$

For the general equation for ghe strain→stress relation, the volumetric strain is

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y)$$

and replacing this in (3):

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_v\delta_{ij}\right) \tag{4}$$